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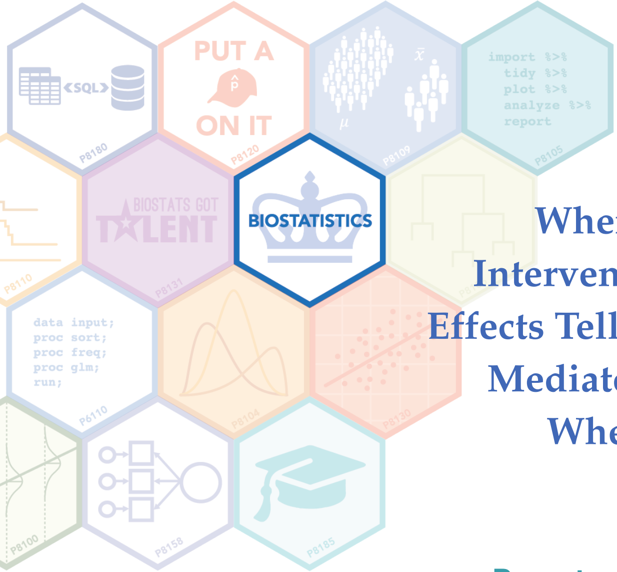

When Randomized Interventional Indirect Effects Tell Stories About Mediated Effects (and When They Don't)

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"True" indirect effects

- **Notation:**
  - $A \in \{a', a\}$ – **exposure (suppose it's randomized)**
  - Y – **outcome**
  - M – **(possible) mediator**
  - $Y(a)$, $Y(m)$, $M(a)$, **etc. – counterfactuals**
- **We would say there is an *individual-level* indirect effect for a given subject if their A affects their M, *and* the resulting change in M affects their Y.**

"True" indirect effects

- **Formally: for subject i,**

$$M_i(a) \neq M_i(a')$$
$$Y_i(m) \neq Y_i(m') \textbf{ for } m = M_i(a) \textbf{ and } m' = M_i(a').$$

- **If there is no individual-level indirect effect for anyone in the population, then any *true* indirect effect measure should be null.**

The natural indirect effect

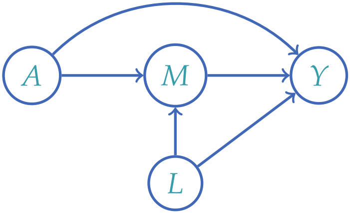
- **The natural indirect effect (NIE) is the most popular causal definition of an indirect effect:**

$$\textbf{NIE} = E[Y\{a, M(a)\}] - E[Y\{a, M(a')\}]$$

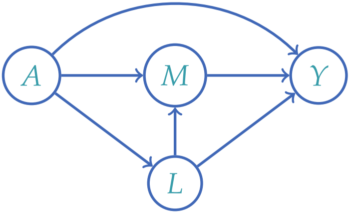
- **Compares counterfactual outcomes under the interventions:**
  - **Set** $A = a$ **and** $M = M(a)$
  - **Set** $A = a$ **and** $M = M(a')$

The natural indirect effect

- **Identification of the NIE relies on several assumptions about confounding.**
- **One is that there is no confounder of the effect of M on Y that's affected by A.**



NIE identified



NIE not identified

Randomized interventional analog of the NIE

- **To circumvent this controversial assumption, VanderWeele et al. (2014) proposed a *randomized interventional analog* of the NIE (NIE^{R}) (previously introduced by Didelez et al. (2006)), which allows for exposure-induced confounding.**
- **Instead of an intervention setting $M = M(a')$, they define a new random variable $G(a')$, with**

  $$G(a') \sim M(a'), \textbf{ but } G(a') \perp\!\!\!\perp M(a').$$

  The NIE^{R} is then defined to be

  $$\textbf{NIE}^{\textbf{R}} = E[Y\{a, G(a)\}] - E[Y\{a, G(a')\}]$$

NIE^{R} lacks the property of a "true" indirect effect

- **Consider the counterfactual distribution:**

$$L(a) = a\varepsilon_L + (1-a)(1-\varepsilon_L)$$
$$M(a,l) = (a+l+al)\varepsilon_M + (1-a)(1-l)(1-\varepsilon_M)$$
$$Y(a,l,m) = (1-a)lm + a(l+m-lm),$$

  where $\varepsilon_L \sim$ Bern(π), $\varepsilon_M \sim$ Bern(β), and $\varepsilon_L \perp\!\!\!\perp \varepsilon_M$.

- **When $\varepsilon_L = 0$, $M(a) = M(a')$; when $\varepsilon_L = 1$, $Y(m) = Y(m')$. Thus, there is no individual-level IE for anyone.**
- **Yet, NIE^{R} $= \pi\{2(1-2\beta)\pi + 2\beta - 1\}$, which is not zero in general, nor is it bounded away from 1 or -1!**

Recovering the "true" indirect effect property

- **Maybe this example seems too contrived. Fair, but we need further assumptions to rule it out.**
- **Under any of the following**
  - $A \not\to L$, $L \not\to M$, **or** $L \not\to Y$
  - $L(a') \perp\!\!\!\perp L(a)$ **(Robins and Richardson, 2010)**
  - **No** L–M **interaction on** Y **on the additive scale (Tchetgen Tchetgen and VanderWeele, 2014),**

  then $\text{NIE}^{\text{R}} = \text{NIE}$, and NIE^{R} is a "true" indirect effect.
- **However, under any of these, the NIE is also identified, so the NIE^{R} provides no advantage.**

Joint stochastic intervention interpretation

- **Despite lacking a true IE interpretation, the NIE^{R} still has a meaningful causal interpretation.**
- **It is the effect comparing two joint stochastic interventions:**
  - **Setting** $A = a$ **and** $M \sim f_{M(a)}(m) = f_{M|A}(m \mid A = a)$
  - **Setting** $A = a$ **and** $M \sim f_{M(a')}(m) = f_{M|A}(m \mid A = a')$

Summary

- **The NIE^R does not have a true IE interpretation without further assumptions.**
- **It does have a meaningful joint stochastic intervention interpretation.**
- **Perhaps there are other assumptions that yield a true IE interpretation while not identifying the NIE.**

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Didelez, V., Dawid, A., and Geneletti, S. (2006). Direct and indirect effects of sequential treatments. In *23rd Annual Conference on Uncertainty in Artifical Intelligence*.

Robins, J. M. and Richardson, T. S. (2010). Alternative graphical causal models and the identification of direct effects. *Causality and Psychopathology: Finding the Determinants of Disorders and Their Cures*, pages 103–158.

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Thank you!

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